Analysis and Synthesis of Coplanar Coupled Lines on Substrates of Finite Thicknesses

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Abstract— **This paper presents two sets of newly developed CAD-oriented formulas for the evaluation of the quasi-static even- and odd-mode characteristics of coplanar coupled lines on substrates of finite thicknesses. The first set of expressions is derived based on the conformal mapping method. Numerical results show that both the even- and odd-mode characteristic impedances and effective dielectric constants computed by these expressions are in good agreement with the results generated by the spectral domain method. The second set of formulas is derived based on a seminumerical approach and can be used to calculate the geometrical parameters of coplanar coupled lines directly from the given electrical parameters, without using an iterative approach. These seminumerical formulas give maximum error in impedance calculation of about 2.0% ($c/h < 2$), over a wide range of impedance and dielectric constant values.**

I. Introduction

Coplanar coupled lines have been widely used in the design of filters and directional couplers [1], [2]. A vast amount of literature has been published on the numerical computation of their characteristic parameters. Wen [1] employed the quasi-static zeroth-order approximation in the study of CPW directional couplers fabricated on substrates of infinite thicknesses. Hatsuda [2] analyzed coplanar coupled lines on substrates of finite thicknesses by the finite difference method. The spectral domain method has also been used for the analysis of coupled lines on single layer substrates [3] as well as on multilayers substrates [4]. However, fast and reliable analytical formulas for calculating the parameters of coplanar coupled lines, which are suitable for CAD tools, are not available in the literature. For these reasons, some newly developed formulas for the evaluation of the quasi-static characteristic parameters of coplanar coupled lines fabricated on substrates of finite thicknesses, are presented here. The first set of expressions is derived based on the conformal mapping method. The second set of formulas is derived based on a seminumerical approach and can be used to calculate the geometrical parameters of coplanar coupled lines directly from the given electrical parameters. Numerical results generated by the spectral domain method are also shown for comparison.

II. Analysis

The configuration to be studied is shown in Fig. 1, where the ground planes are assumed to be infinitely wide. All conductors are assumed to be infinitely thin and perfectly conducting. The two coupled strips, of width $(b - a)$, are placed between two ground planes, of spacing $2c$, which are located on a substrate of thickness h, with relative permittivity ε_r. The two coupled strips are separated by a distance of $2a$. It is assumed that the air-dielectric interfaces, where all the conductors are located, can be dealt with as though perfect magnetic walls are present in them. This structure supports two fundamental modes, namely odd and even. The total capacitance per unit length is considered as the sum of the capacitances in the upper region (air) and lower region (air and dielectric layer). The lower region capacitance is evaluated by the approximate technique suggested in [5] as the sum of the free-space capacitance in the absence of the dielectric and the capacitance of the dielectric layer, assumed to have permittivity $(\varepsilon_r - 1)$.

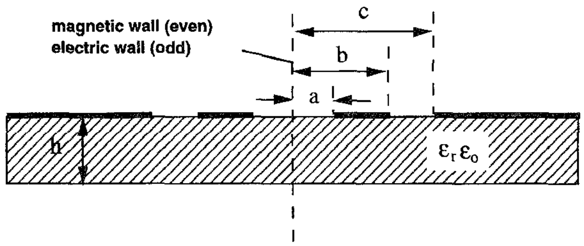

Fig. 1. Coplanar coupled line.

A. Even-Mode

The even-mode is analyzed by assuming a magnetic wall is present at the center of the structure. The even-mode capacitance per unit length is obtained through a sequence of conformal mappings [5] and the resulting formulas can be derived as

$$C_e(\varepsilon_r) = 2\varepsilon_o \frac{K(k_{e1})}{K'(k_{e1})} + (\varepsilon_r - 1)\varepsilon_o \frac{K(k_{e2})}{K'(k_{e2})} \tag{1a}$$

$$k_{e1}^2 = \frac{b^2 - a^2}{c^2 - a^2} \tag{1b}$$

$$k_{e2}^2 = \frac{\sinh^2(\pi b/2h) - \sinh^2(\pi a/2h)}{\sinh^2(\pi c/2h) - \sinh^2(\pi a/2h)} \tag{1c}$$

where $K(k)$ and $K'(k)$ are the complete elliptic integral of the first kind and its complement. Accurate expressions for the ratio $K(k)/(K'(k)$ are available in [6].

B. Odd-Mode

The odd-mode is analyzed by assuming an electric wall is present at the center of the structure. The lower region capacitance is obtained through the sequence of transformations depicted in Fig. 2. The configuration in Fig. 2(a) is first converted into the one shown in Fig. 2(c) by the following expressions

$$\frac{W}{H} = \frac{K(k_{c1})}{K'(k_{c1})} = \alpha \tag{2a}$$

$$\frac{W_1}{W} = \frac{F\left(\arcsin\sqrt{\frac{\sinh^2(\pi c/2h) - \sinh^2(\pi a/2h)}{\sinh^2(\pi c/2h)\cosh^2(\pi a/2h)}}, k_{c1}\right)}{K(k_{c1})} = \beta_1 \tag{2b}$$

$$\frac{W_2}{W} = \frac{F\left(\arcsin\sqrt{\frac{\sinh^2(\pi c/2h) - \sinh^2(\pi a/2h)}{\sinh^2(\pi c/2h)}}, k_{c1}\right)}{K(k_{c1})} = \beta_2 \tag{2c}$$

$$k_{c1}^2 = \frac{\sinh^2(\pi c/2h)}{\sinh^2(\pi b/2h)} \frac{\sinh^2(\pi b/2h) - \sinh^2(\pi a/2h)}{\sinh^2(\pi c/2h) - \sinh^2(\pi a/2h)} \tag{2d}$$

where $F(\phi, k)$ is the incomplete elliptic integral of the first kind, written in Jacobi's notation. A magnetic wall is then assumed to be present at the center of the slot, and hence the structure in Fig. 2(c) can be considered as the sum of two capacitances, as depicted in

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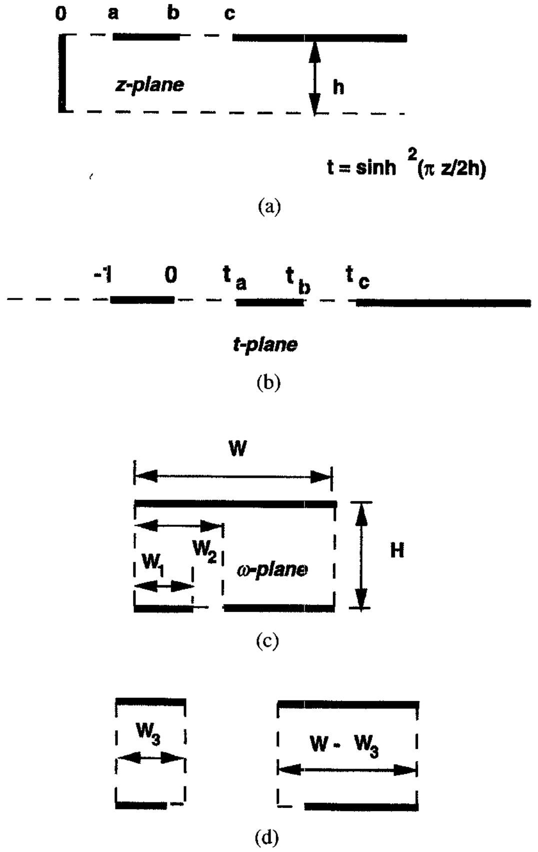


Fig. 2. Conformal transformations for odd-mode capacitance in the lower region.

Fig. 2(d). In result, the total odd-mode capacitance per unit length is given by

$$C_o(\varepsilon_r) = 2\varepsilon_o \frac{K(k_{o1})}{K'(k_{o1})} + (\varepsilon_r - 1)\varepsilon_o \cdot \left(\frac{K(k_{o2})}{K'(k_{o2})} + \frac{K(k_{o3})}{K'(k_{o3})}\right) \tag{3a}$$

$$k_{o1}^2 = \frac{c^2(b^2 - a^2)}{b^2(c^2 - a^2)} \tag{3b}$$

$$\frac{F(\arcsin(k_{o2}/k_{c2}), k_{c2})}{K(k_{c2})} = \frac{W_1}{W_3} = \beta_1/\beta_3 \tag{3c}$$

$$\frac{F(\arcsin(k_{o3}/k_{c3}), k_{c3})}{K(k_{c3})} = \frac{W - W_2}{W - W_3} = \frac{1 - \beta_2}{1 - \beta_3} \tag{3d}$$

$$\frac{K(k_{c2})}{K'(k_{c2})} = \frac{W_3}{H} = \alpha\beta_3 \tag{3e}$$

$$\frac{K(k_{c3})}{K'(k_{c3})} = \frac{W - W_3}{H} = \alpha(1 - \beta_3) \tag{3f}$$

$$\beta_3 = \frac{W_3}{W} = \frac{W_1 + W_2}{2W} = \frac{\beta_2 + \beta_1}{2}. \tag{3g}$$

Expressions (2b), (2c), (3c), and (3d) can be evaluated using either the approximated formulas reported in [7], or the routines in [8]. A solution for (3e) and (3f) is given in the Appendix. Hence, the coupler odd- and even-mode characteristic impedances and relative effective dielectric constants can be calculated by the well-known equations

$$\varepsilon_{\text{eff}(o,e)} = \frac{C_{(o,e)}(\varepsilon_r)}{C_{(o,e)}(1)} \tag{4a}$$

$$Z_{o(o,e)} = c_v^{-1}\sqrt{C_{(o,e)}(\varepsilon_r)C_{(o,e)}(1)} \tag{4b}$$

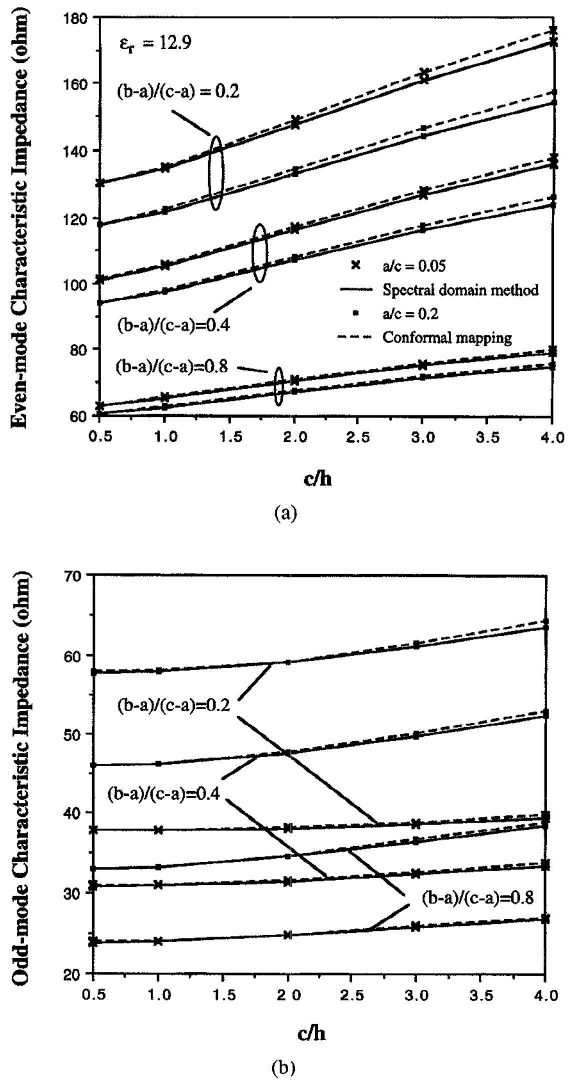


Fig. 3. Characteristic impedance evaluation of coplanar coupled lines. (a) Even-mode. (b) Odd-mode.

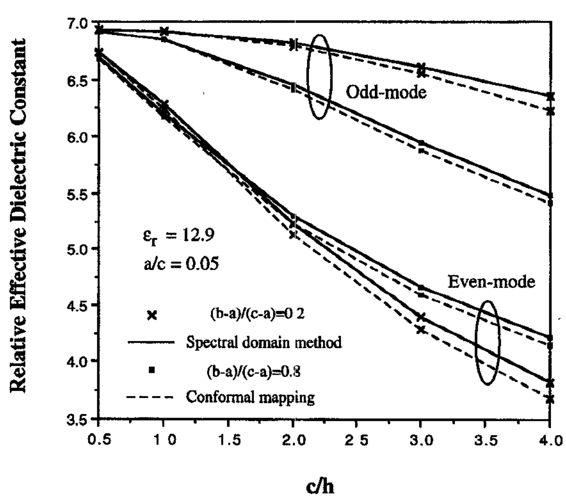


Fig. 4. Effective dielectric constant evaluation of coplanar coupled lines.

where c_v is the velocity of light in vacuum. The expressions derived in this section can easily be extended to coplanar coupled lines with upper shielding [5].

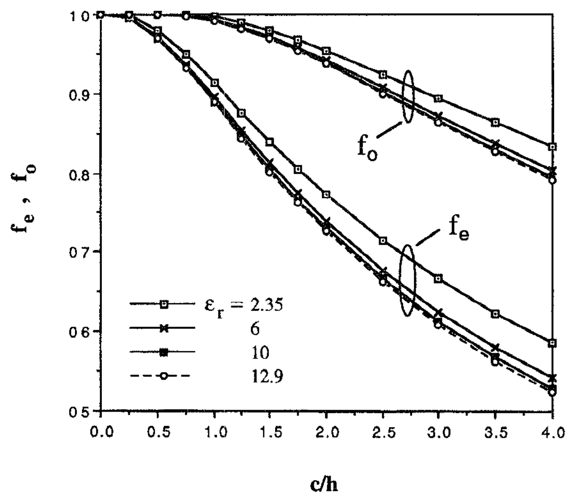


Fig. 5 Plot of function f_o & f_e versus c/h and ε_r.

III. Synthesis

The design procedure of couplers usually requires the geometrical parameters (a) and (b) to be determined from the input values such as the even- and odd-mode characteristic impedances, substrate thicknesses, dielectric constants, and ground-plane spacing. However, in obtaining the physical dimensions from the given electrical parameters using analyses mentioned above is involved since the expressions derived can only be solved through an iterative approach. For this reason, some quick and accurate formulas have been devised for coplanar coupled line synthesis. The seminumerical formula proposed here for the evaluation of the even- and odd-mode characteristic impedances is

$$Z_{o(o,e)} = \frac{120\pi K'(k_{(o1,e1)})/K(k_{(o1,e1)})}{\sqrt{2[2+f_{(o,e)}(\varepsilon_r-1)]}} \tag{5}$$

where $f_{o,e}$ are some functions of ε_r and c/h. The optimum values of $f_{o,e}$ are defined as the quantities that give the minimum relative error between the sets of impedance values calculated by (5) and by the spectral domain approach, for the chosen values of ε_r and c/h (which will be discussed further in Section IV). This step can be achieved easily by any computer searching algorithm. Consequently, the synthesis equations are obtained by solving (5) and the resulting expressions are simply

$$\frac{K(t_{(o,e)})}{K'(t_{(o,e)})} = \frac{120\pi}{Z_{o(o,e)}\sqrt{2[2+f_{(o,e)}(\varepsilon_r-1)]}} \tag{6a}$$

$$b = \frac{t_e}{t_o}c \tag{6b}$$

$$a = b\sqrt{\frac{1-t_o^2}{1-t_e^2}}. \tag{6c}$$

IV. Numerical Computation and Results

In our first example, numerical results are generated to examine the accuracy of the analytical expressions derived in Section II. Figs. 3 and 4 show the plot of the even- and odd-mode characteristic impedance and effective dielectric constant values ($\varepsilon_r = 12.9$), evaluated as a function of a/c, $(b-a)/(c-a)$, and c/h. For purposes of comparison, values obtained by the spectral domain approach [9] are also shown in the diagram. Upon examining also sets of data (not shown here) based on different dielectric constant values ($\varepsilon_r = 2.35$, 6.0, 10.0), it may be concluded that both the even- and odd-mode characteristic impedance and effective dielectric constant evaluation is of sufficient precision for most of the applicable range of physical dimensions used and available dielectric materials.

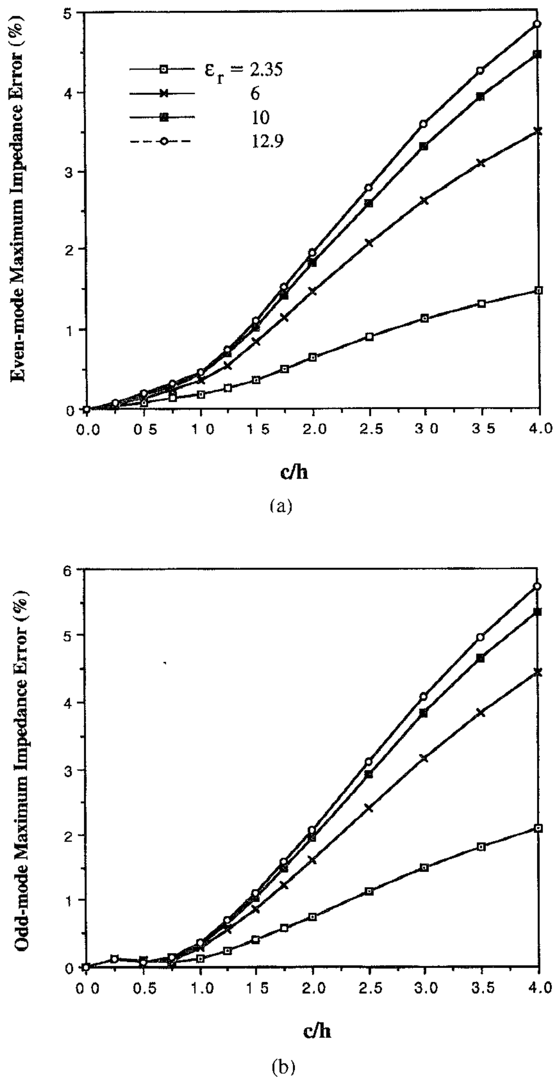
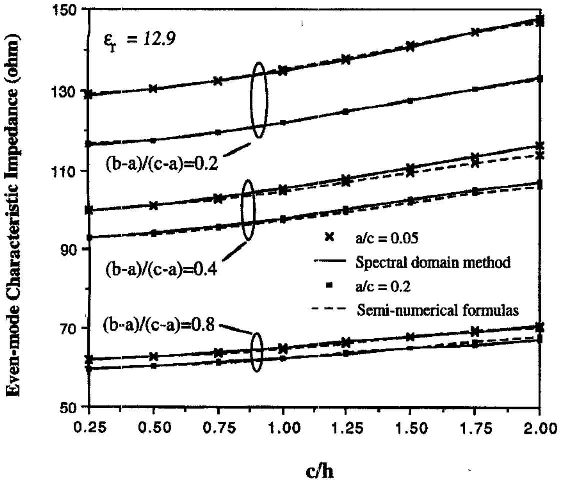


Fig. 6. Maximum impedance error of the seminumerical formulas versus c/h and ε_r. (a) Even-mode. (b) Odd-mode

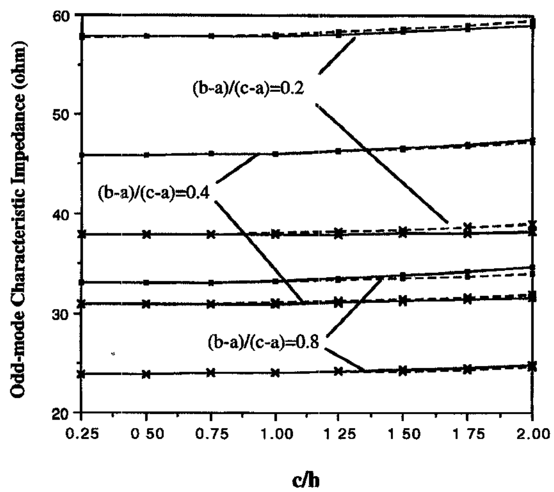
In our next example, the accuracy of the seminumerical formulas in (5) will be demonstrated. As mentioned previously, the optimum values of f_e and f_o are determined by minimizing the error between the two sets of impedance values obtained by the seminumerical formulas and by the spectral domain approach. The relationship between (f_e and f_o) and (ε_r and c/h) thus obtained is shown in Fig. 5. The set of values that were used to obtain these curves are

$$\varepsilon_r = 2.35, 6.0, 10.0, 12.9$$
$$c/h = 0.25, 0.5, 0.75, \cdots, 4.0$$
$$a/c = 0.05, 0.1, 0.2$$
$$\frac{b-a}{c-a} = 0.2, 0.4, 0.8.$$

In Fig. 6, the corresponding maximum percentage error of the proposed formulas in both the even- and odd-mode impedance evaluation is shown. It can easily be seen that the calculated error increases with both ε_r and c/h. To further illustrate the accuracy of the formulas, comparison has also been made ($\varepsilon_r = 12.9$) with respect to the even and odd-mode characteristic impedance values versus different geometrical parameters, as depicted in Fig. 7. Expressions



(a)



(b)

Fig. 7. Characteristic impedance evaluation by the spectral domain method and by the seminumerical formulas. (a) Even-mode. (b) Odd-mode.

used in approximating f_e and f_o (c/h and 2) in these calculations are

$$\begin{aligned} f_e &= 1 - \frac{\varepsilon_r}{\varepsilon_r + 0.641}\{-0.0175(c/h) + 0.18(c/h)^2 \\ &\quad - 0.05(c/h)^3\} \\ f_o &= 1 - \frac{\varepsilon_r}{\varepsilon_r + 1.345}\{-0.0177(c/h) + 0.0264(c/h)^2\}. \end{aligned} \tag{7}$$

Note that the discrepancies between the values calculated by (7) and those shown in Fig. 5 are less than 0.5%. It can also be observed from Fig. 6 that the discrepancies between the impedance values obtained by the seminumerical formulas and by the spectral domain method are less than 2.0% ($c/h < 2$) for both even- and odd-mode impedance computations.

V. Conclusion

Some closed-form expressions have been devised for obtaining the quasi-static even- and odd-mode characteristic parameters of coplanar coupled lines. Numerical results generated by these formulas have shown excellent agreements with reference data obtained by the spectral domain method. A set of synthesis formulas has been proposed for the direct computation of the geometrical parameters of coplanar coupled lines, and the maximum impedance error thus obtained depends largely upon the ratio of the ground plane spacing to the substrate thicknesses. These new expressions make possible fast calculations of the parameters of coplanar coupled lines with precision sufficient for most engineering applications, thus making it an excellent choice for use in CAD-oriented design tools.

Appendix

The general form of the expressions given in (3e), (3f), and (6a) is

$$\frac{K(\alpha)}{K'(\alpha)} = \beta. \tag{A1}$$

An approximated solution for the above equation is

$$\alpha = \begin{cases} \left(\dfrac{e^{\pi\beta} - 2}{e^{\pi\beta} + 2}\right)^2, & 1 < \beta < \infty \\ \sqrt{1 - \left(\dfrac{e^{\pi/\beta} - 2}{e^{\pi/\beta} + 2}\right)^4}, & 0 < \beta < 1 \end{cases}. \tag{A2}$$

References

[1] C. P. Wen, "Coplanar-waveguide directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, no. 6, pp. 318–322, June 1970.

[2] T. Hatsuda, "Computation of coplanar-type strip-line characteristics by relaxation method and its application to microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, no. 10, pp. 795–802, Oct. 1975.

[3] T. Kitazawa and R. Mittra, "Quasi-static characteristics of asymmetrical and coupled coplanar-type transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, no. 9, pp. 771–778, Sept. 1985.

[4] R. R. Boix and M. Horno, "Modal quasistatic parameters for coplanar multiconductor structures in multilayered substrates with arbitrary transverse dielectric anisotropy," *IEE Proc.*, pt. H, vol. 136, no. 1, pp. 76–79, Feb. 1989.

[5] G. Ghione and C. Naldi, "Coplanar waveguides for MMIC applications: effect of upper shielding, conductor backing, finite-extent ground planes, and line-to-line coupling," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, no. 3, pp. 260–267, Mar. 1987.

[6] W. Hilberg, "From approximations to exact relations for characteristic impedances," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, no. 3, pp. 259–265, May 1969.

[7] K. K. M. Cheng and I. D. Robertson, "Quasi-TEM study of microshield lines with practical cavity sidewall profiles," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 12, Dec. 1995.

[8] W. Press, B. Flannery, S. Teukolsky, and W. Vetterling, *Numerical Recipes, The Art of Scientific Computing*. Cambridge, England: Cambridge Univ. Press, 1986.

[9] K. K. M. Cheng and I. D. Robertson, "Numerically efficient spectral domain approach to the quasi-TEM analysis of supported coplanar waveguide structures," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 10, pp. 1958–1965, Oct. 1994.